Question
Calculate

$$J = \int_0^1 \frac{\cos x}{1+x^2}\, dx$$

by using

(i) the trapezium rule with 7 ordinates,

(ii) Simpson's rule with 7 ordinates.

Compare your answers with the exact result $J = 0.6829....$

Answer

$$J = \int_0^1 \frac{\cos x}{(1+x^2)}\, dx$$

(i) Trapezium rule with 7 ordinates:

$J \approx \frac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + 2y_6 + y_7)$

where $d = \frac{1-0}{7-1} = \frac{1}{6}$

$x_1 = 0 \quad x_4 = \frac{3}{6}$
$x_2 = \frac{1}{6} \quad x_5 = \frac{4}{6}$
$x_3 = \frac{2}{6} \quad x_6 \frac{5}{6}$
$x_7 = 2$

$$y_i = f(x_i); \quad f(x) = \frac{\cos x}{1+x^2}$$

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
y	1.0	0.95949	0.85046	0.70207	0.54408	0.39683	0.27015

$J \approx \frac{1}{12}(1.0 + 2 \times [0.95949 + 0.85046 + 0.70207 + 0.54408 + 0.39683] + 0.27015)$

$$\begin{aligned} J &\approx \frac{1}{12}(1.27015 + 2 \times 3.45293) \\ &= \frac{1}{12}(1.27015 + 6.90586) \\ &= \frac{1}{12}(8.17601) \\ &= \underline{0.681334...} \end{aligned}$$

(ii) Simpson's rule with 7 ordinates:

$J \approx \frac{d}{2}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$

6 equal segments $\Rightarrow h = \frac{1}{6}$

so we have the same y_i as above.

Hence

$$\begin{aligned} J &\approx \frac{1}{18}(1.0 + 4 \times (0.95949 + 0.70207 + 0.39683) \\ &\quad +2 \times (0.85046 + 0.54408) + 0.27015) \\ &= \frac{1}{18}(1.27015 + 0.823356 + 2.78908) \\ &= \frac{1}{18} \times 12.2928 \\ &= \underline{0.682933...} \end{aligned}$$

Actual=0.6829 to 4sf

(i) is accurate to 0.2% or 2sf

(ii) is accurate to 4sf