**Question**
For each of the following series, determine the values of $x$ for which the series converges.

1. $\sum_{n=1}^{\infty}((x+2)/(x-1))^n/(2n-1)$;
2. $\sum_{n=1}^{\infty} 1/((x+n)(x+n-1))$;

**Answer**
We can use the same techniques that we have developed for power series for other series, that are not strictly speaking power series. For instance, we can apply the ratio test to the series, for all the values of $x$ for which the terms are defined.

1. first, we note that this series is not defined at $x=1$, but is defined for all other values of $x$. Applying the ratio test, we calculate:
$$\lim_{n\to\infty}\left|\frac{((x+2)/(x-1))^{n+1}/(2(n+1)-1)}{((x+2)/(x-1))^n/(2n-1)}\right| = \frac{|x+2|}{|x-1|}\lim_{n\to\infty}\frac{2n-1}{2n+1} = \frac{|x+2|}{|x-1|}.$$
Hence, this series converges absolutely for $\frac{|x+2|}{|x-1|}<1$, that is for $|x+2|<|x-1|$, which is the open ray $(-\infty,-\frac{1}{2})$, and diverges for $\frac{|x+2|}{|x-1|}>1$, which is the union $(-\frac{1}{2},1)\cup(1,\infty)$.

   At $x=-\frac{1}{2}$, the only remaining point at which to test for convergence, the series becomes
$$\sum_{n=1}^{\infty}\frac{1}{2n-1}\left(\frac{-\frac{1}{2}+2}{-\frac{1}{2}-1}\right)^n = \sum_{n=1}^{\infty}\frac{1}{2n-1}(-1)^n,$$
which converges conditionally, by the alternating series test. Hence, the series converges on the closed ray $(-\infty,-\frac{1}{2}]$.

2. for this series, first note that the series is not defined at $x=0$, $x=1$, $x=2$, et cetera, and so the domain of consideration is the complement in $\mathbf{R}$ of the non-negative integers $\mathbf{W}=\{0,1,2,\ldots\}$ (the **whole numbers**). Applying the ratio test, we calculate
$$\lim_{n\to\infty}\left|\frac{1/((x+n+1)(x+n+1-1))}{1/((x+n)(x+n-1))}\right| = \lim_{n\to\infty}\left|\frac{x+n-1}{x+n+1}\right| = 1$$
for every (allowable) value of $x$, and so yields no information. However, we are saved by the observation that the series
$$\sum_{n=1}^{\infty}\frac{1}{(n+\alpha)(n+\beta)}$$

converges for all $\alpha$, $\beta$, by limit comparison to the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Hence, taking $\alpha = x$ and $\beta = x - 1$, we have that $\sum_{n=1}^{\infty} 1/((x+n)(x+n-1))$ converges at every value of $x$ for which it is defined, namely the union

$$(-\infty, 0) \cup (0, 1) \cup (1, 2) \cup (2, 3) \cup \cdots = \mathbf{R} - \mathbf{W}.$$