QUESTION Show that any real cubic polynomial can be expressed in the form

$$\alpha x^{(3)} + \beta x^{(2)} + \gamma x^{(1)} + \delta x^{(0)}$$

where $x^{(3)} = x(x-1)(x-2)$.

ANSWER

$$ax^3 + bx^2 + cx + d = ax^{(3)} + (3a+b)x^{(2)} + (a+b+c)x^{(1)} + d$$