Question
Find the general solution of the following differential equations

(i) $\cos y - x \sin y \dfrac{dy}{dx} = 0$

(ii) $xy + x\dfrac{dy}{dx} = e^x$

Answer

(i) $\cos y - x \sin y \dfrac{dy}{dx} = 0$

Not variables separable. Not homogeneous (sin's and cos's)

Consider

$$\frac{d}{dx})x \cos y) = \cos y - x \sin y \frac{dy}{dx}$$

i.e., the LHS is an exact derivative. Thus we can rewrite the equation as
$\dfrac{d}{dx}(x \cos y) = 0$
$\Rightarrow \underline{x \cos y = c}$ where c is constant

(ii) $xy + x\dfrac{dy}{dx} = e^x$

Not variables separable. Not homogeneous (e^x's)

Consider

$$\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$$

i.e., the LHS is an exact derivative. Thus we can rewrite the equation as

$$\begin{aligned} \frac{d}{dx}(xy) &= e^x \\ \Rightarrow \quad xy &= \int e^x dx = e^x + c \end{aligned}$$

$\Rightarrow \underline{xy = e^c + c}$