Question Find the general solution of the following differential equations

(i) $\dfrac{dy}{dx} = \sin x$

(ii) $e^x \dfrac{dy}{dx} = \sin x$

(iii) $\dfrac{dy}{dx} = 1 + y$

(iv) $\dfrac{dy}{dx} = 1 + y^2$

Answer

(i)

$\dfrac{dy}{dx} = \sin x$ is of the form $\dfrac{dy}{dx} = f(x)$, type (i).
Solve by direct integration:

$\Rightarrow \int dy = \int \sin x \, dx$

$\Rightarrow \underline{y = -\cos x + c}$

(ii)

$e^x \dfrac{dy}{dx} = \sin x.$

Rearrange to get $\dfrac{dy}{d} = e^{-x} \sin x$. Again of the form $\dfrac{dy}{dx} = f(x)$, type (i).

Thus

$\int dy = \int e^{-x} \sin x dx$

integrate by parts with $\left\{ \begin{array}{ll} u = e^{-x} & \frac{dv}{dx} = \sin x \\ \frac{du}{dx} = -e^{-x} & v = -\cos x \end{array} \right.$

$\Rightarrow y = -e^{-x} \cos x - \int e^{-x} \cos x dx$

integrate by parts with $\left\{ \begin{array}{ll} u = e^{-x} & \frac{dv}{dx} = \cos x \\ \frac{du}{dx} = -e^{-x} & v = \sin x \end{array} \right.$

$\Rightarrow y = -e^{-x} \cos x - [e^{-x} \sin x] + \int -e^{-x} \sin x dx$

$\Rightarrow y = -e^{-x}(\cos x + \sin x) - y + c$

$\Rightarrow \underline{y = -\dfrac{e^{-x}}{2}(\cos x + \sin x) + c'}$

(iii)

$\dfrac{dy}{dx} = 1 + y$ is of the form $\dfrac{dy}{dx} = f(y)$ so rearrange to get

$$\begin{aligned} & \int \frac{dy}{1+y} = \int dx \\ \Rightarrow \quad & \ln(1+y) = x + c \\ \Rightarrow \quad & 1 + y = e^{x+c} \\ \Rightarrow \quad & \underline{y = e^{x+c} - 1} \end{aligned}$$

(iv)

$\frac{dy}{dx} = 1 + y^2$ is of the form $\frac{dy}{dx} = f9y)$ so rearrange to get

$$\begin{aligned} & \int \frac{dy}{1+y^2} = \int dx \\ \Rightarrow \quad & \arctan y = x + c \\ \Rightarrow \quad & \underline{y = \tan(x + c)} \end{aligned}$$