Question

An AC current I, in a circuit with inductance L and resistance R is given by

$$L\frac{dI}{dt} + RI = E,$$

where $L,\ R$ and E are constant. Find I, given that $I = 0$ when $t = 0$.

Answer

$L\frac{dI}{dt} + RI + E$

Could do by variables separable method or

$\frac{dI}{dt} + \frac{R}{L}I = \frac{E}{L}$ cf $\frac{dI}{dt} + PI = Q$

Use $P = \frac{R}{L},\ Q = \frac{E}{L}$.

Thus integrating factor

$R = \exp\left(\int P\,dt\right) = \exp\left(\int \frac{R}{L}\,dt\right) = e^{\frac{R}{L}t}$

$\Rightarrow e^{\frac{R}{L}t}\frac{dI}{dt} + \frac{R}{L}e^{\frac{R}{L}t}I = \frac{E}{L}e^{\frac{R}{L}t}$

$\Rightarrow \frac{d}{dt}\left(e^{\frac{R}{L}t}I\right) = \frac{E}{L}e^{\frac{R}{L}t}$

$\Rightarrow e^{\frac{R}{L}t}I = \frac{E}{L}e^{\frac{R}{L}t}$

$\Rightarrow e^{\frac{R}{L}t}I = \frac{E}{L}\frac{L}{R}e^{\frac{R}{L}t} + c$

$\Rightarrow I = \frac{E}{R} + ce^{\frac{R}{L}t}$

where c is constant

Now if $I = 0$ when $t = 0$

$0 = \frac{E}{R} + ce^0 \Rightarrow c = -\frac{E}{R}$

Thus $\underline{I = \frac{E}{R}\left(1 - e^{\frac{R}{L}t}\right)}$