QUESTION

(a) Solve the following linear programming problem using the simplex method.

$$\begin{array}{ll} \text{Maximize} & z = -18x_1 + 4x_2 + 5x_3 \\ \text{subject to} & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \\ & 8x_1 - 3x_2 + 3x_3 \leq 21 \\ & 3x_1 + 2x_2 + x_3 = 6 \\ & -2x_1 + 4x_2 + 3x_3 \geq 15. \end{array}$$

(b) A lawnmower manufacturer produces traditional (non-powered), electric and petrol models. Demand for the next two months is shown in the following table.

Model	Month 1	Month 2
Traditional	150	200
Electric	600	800
Petrol	200	250

For each lawnmower, the production cost, the time required by the labour force for manufacture and the time required by the labour force for assembly are shown in the following table; the current inventory levels (at the start of month 1) are also listed.

Model	Production cost (£)	Time for manufacture (hours)	Time for assembly (hours)	Current inventory
Traditional	20	3	5	30
Electric	30	5	8	50
Petrol	45	6	9	20

Last month, the company used a total of 13 000 hours of labour. The company's labour relations policy will not allow the combined total hours of labour (manufacture plus assembly) to increase or decrease by more than 500 hours from month to month.

There are end-of-month inventory holding costs. For each lawnmower in stock at the end of a month, the holding cost is 3% of its production cost. The company requires at least 25 lawnmowers of each model to be in stock at the end of the second month.

Write down a linear programming formulation (but do not attempt to solve it) for the problem of planning production so that demand is satisfied at minimum total cost. You may ignore any requirements for variables to be integer-valued.

ANSWER

(a)

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	8	−3	3	1	0	0	0	21
a_1	0	0	3	2	1	0	0	1	0	6
a_2	0	0	−2	4	3	0	−1	0	1	15
	1							1	1	0
	1	0	−1	−6	−4	0	1	0	0	−12
	0	1	18	−4	−5	0	0	0	0	0

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	$\frac{25}{2}$	0	$\frac{9}{2}$	1	0	$\frac{3}{2}$	0	30
x_2	0	0	$\frac{3}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	3
a_2	0	0	−8	0	1	0	−1	−2	1	3
	1	0	8	0	−1	0	1	3	0	−3
	0	1	24	0	−3	0	0	2	0	12

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	$\frac{97}{2}$	0	0	1	$\frac{9}{2}$	$\frac{21}{2}$	$-\frac{9}{2}$	$\frac{33}{2}$
x_2	0	0	$\frac{11}{2}$	1	0	0	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
x_3	0	0	−8	0	1	0*−1	−2	1	3	
	1	0	0	0	0	0	0	1	1	0
	0	1	0	0	0	0	−3	−4	3	21

Phase 1 end: discard z', a_1, a_2.

Basic	z	x_1	x_2	x_3	s_1	s_2	
s_1	0	−1	−9	0	1	0	3
s_2	0	11	2	0	0	1	3
x_3	0	3	2	1	0	0	6
	1	33	6	0	0	0	30

Thus, an optimal solution is

$$x_1 = 0 \; x_2 = 0 \; x_3 = 6 \; z = 30$$

(b) Let $x_i,\ y)i,\ z_i$ be the production of traditional, electric and petrol lawnmowers in month i, for $i = 1, 2$.

Let $s_i,\ t_i,\ u_i$ be the stocks of traditional, electric and petrol lawnmowers in month i, for $u = 1, 2$

Let $a_i,\ b_i$ be the increase and decrease in hours of labour from month $i - 1$ to i, for $i = 1, 2$.

Let l_i be the labour hours in month i, for $i = 1, 2$.

Maximize	$z = 20(x_1 + x_2) + 30(y_1 + y_2) + 45(z_1 + z_2)$
	$+0.6(s_0 + s_1 + s_2) + 0.9(t_0 + t_1 + t_2) + 1.35(u_0 + u_1 + u_2)$
Subject to	$x_i \geq 0, y_i \geq 0, z_i \geq 0, i = 1, 2$
	$s_i \geq 0, t_i \geq 0, u_i \geq 0, i = 1, 2$
	$l_i \geq 0, a_i \geq 0, b_i \geq 0, i = 1, 2$

$$
\begin{aligned}
s_0 &= 30 \\
s_0 + x_1 - s_1 &= 150 \\
s_3 + x_2 - s_2 &= 200 \\
s_2 &\geq 25 \\
t_0 &= 50 \\
t_0 + y_1 - t_1 &= 600 \\
t_1 + y_2 - t_2 &= 800 \\
t_2 &\geq 25 \\
u_0 &= 20 \\
u_0 + z_1 - u_1 &= 200 \\
u_1 + z_2 - u_2 &= 250 \\
u_2 &\geq 25 \\
l_1 &= 8x_1 + 13y_1 + 15z_1 \\
l_2 &= 8x_2 + 13y_2 + 15z_2 \\
l_1 &= l_0 + a_1 - b_1 \\
l_2 &= l_1 + a_2 - b_2 \\
a_1 &\leq 500 \\
b_1 &\leq 500 \\
a_2 &\leq 500 \\
b_2 &\leq 500 \\
l_0 &= 4000
\end{aligned}
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