Question

Obtain a fundamental matrix $\Phi(t)$ for each of the systems which satisfies the initial conditions $\Phi(0) = I$, and hence solve the above problems with the initial conditions

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(a) $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

(a) $\begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$

(e) $\begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$

(f) $\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

(g) $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

Answer

(a)

$$\Phi(t) = \begin{pmatrix} -1/3e^{-t} + 4/3e^{2t} & 2/3e^{-t} - 2/3e^{2t} \\ -2/3e^{-t} + 2/3e^{2t} & 4/3e^{-t} - 1/3e^{2t} \end{pmatrix}$$

(b)

$$\Phi(t) = \begin{pmatrix} 3 - 2e^{-2t} & -3/2 + 3/2e^{-2t} \\ 4 - 4e^{-2t} & -2 + 3e^{-2t} \end{pmatrix}$$

(c)

$$\Phi(t) = \begin{pmatrix} 3/2e^{t} - 1/2e^{-t} & -1/2e^{t} + 1/2e^{-t} \\ 3/2e^{t} - 3/2e^{-t} & -1/2e^{t} + 3/2e^{-t} \end{pmatrix}$$

(d)

$$\Phi(t) = \begin{pmatrix} 1/5e^{-3t} + 4/5e^{2t} & -1/5e^{-3t} + 1/5e^{2t} \\ -4/5e^{-3t} + 4/5e^{2t} & 4/5e^{-3t} + 1/5e^{2t} \end{pmatrix}$$

(e)

$$\Phi(t) = \begin{pmatrix} \cos t + 2\sin t & -5\sin t \\ \sin t & \cos t - 2\sin t \end{pmatrix}$$

(f)

$$\Phi(t) = \begin{pmatrix} e^{-t}\cos 2t & -2e^{-t}\sin 2t \\ 1/2e^{-t}\sin 2t & e^{-t}\cos 2t \end{pmatrix}$$

(g)

$$\Phi(t) = \begin{pmatrix} -1/2e^{2t} + 3/2e^{4t} & 1/2e^{2t} - 1/2e^{4t} \\ -3/2e^{2t} + 3/2e^{4t} & 3/2e^{2t} - 1/2e^{4t} \end{pmatrix}$$

(h)

$$\Phi(t) = \begin{pmatrix} 2te^{t} + e^{t} & -4te^{t} \\ te^{t} & e^{t} - 2te^{t} \end{pmatrix}$$