Question
Find the general solution of the differential equation:

$$\frac{d^2x}{dt^2} + 4x = 1 + \sin 2t$$

Answer
Need general solution of

$$\frac{d^2x}{dt^2} + 4x = 1 + \sin 2t$$

Complementary Function: $m^2 + 4 = 0 \Rightarrow m = \pm 2j$
Hence the general solution is $\Rightarrow x_c(t) = A\cos 2t + B\sin 2t$
For the particular integral for $\dfrac{d^2x_1^*}{dt^2} + 4x_1^* = 1$
Try $x_1^* = c \Rightarrow \dfrac{d^2x_1^*}{dt^2} = 0 \Rightarrow 4c = 1 \Rightarrow c = \dfrac{1}{4}$
Find a particular integral for $\dfrac{d^2x_2^*}{dt^2} + 4x_2^* = \sin 2t$
We can not try the obvious $x_2^* = P\cos 2t + Q\sin 2t$ because this is the same as the complementary function. So instead we will try
$x_2^* = t(P\cos 2t + Q\sin 2t)$
then we shall try to find P and Q

$$\begin{aligned}
\frac{dx_2^*}{dt} &= (P\cos 2t + Q\sin 2t) + t(-2P\sin 2t + 2Q\cos 2t) \\
&= (2Qt + P)\cos 2t + (-2Pt + Q)\sin 2t \\
\frac{d^2x_2^*}{dt^2} &= (-4Pt + 2Q + 2Q)\cos 2t + (-4Qt - 2P - 2P)\sin 2t \\
\frac{d^2x_2^*}{dt^2} + 4x_2^* &= 4\{\cos 2t(Q - Pt + Pt) + \sin 2t(-Qt - P + Qt)\} \\
&\equiv \sin 2t
\end{aligned}$$

Hence $Q = 0$ and $-4P = 1 \Rightarrow P = -\frac{1}{4}$
Hence$x_2^* = -\frac{1}{4}t\cos 2t$
The General Solution is:

$$\begin{aligned}
x &= x_c + x_1^* + x_2^* \\
&= \left(A - \frac{1}{4}t\right)\cos 2t + B\sin 2t + \frac{1}{4}
\end{aligned}$$