

**Question**

Find the general solution of the differential equation:

$$4\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + x = e^{-t}$$

**Answer**

$$4\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + x = e^{-t}$$

Complementary Function:

auxiliary equation  $4m^2 - 4m + 1 = 0 \Rightarrow (2m - 1)^2 = 0$

TWO solutions with  $m = \frac{1}{2}$

Hence the Complementary Function is  $x_c = (At + B)e^{\frac{1}{2}t}$

Particular integral method of undetermined coefficients

$$\begin{aligned} \text{Let } x^* &= Ce^{-t} \\ 4\frac{d^2x^*}{dt^2} - 4\frac{dx^*}{dt} + x &= Ce^{-t}[4 + 4 + 1] \equiv e^{-t} \end{aligned}$$

$$\text{Hence } C = \frac{1}{9}$$

Hence the general solution is now

$$x = (At + B)e^{\frac{1}{2}t} + \frac{1}{9}e^{-t}$$