Question

Find the particular integral for the differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 1 + 2t + t^2$$

Answer

There are many Particular Integrals, but we use the method of undetermined coefficients.

Let

$$
\begin{aligned}
x^* &= a_0 + a_1 t + a_2 t^2 \\
\frac{dx^*}{dt} &= a_1 + 2a_2 t \\
\frac{d^2x^*}{dt^2} &= 2a_2
\end{aligned}
$$

We try to fix a_0, a_1, a_2

Substitute into equation

$$
\begin{aligned}
\frac{d^2x^*}{dt^2} + 3\frac{dx^*}{dt} + 2x &= 1 + 2t + t^2 \qquad (A)\\
&= 2a_2 + 3(a_1 + 2a_2 t) + 2(a_0 + a_1 t + a_2 t^2) \\
&= [2a_2 + 3a_1 + 2a_0] + [6a_2 + 2a_1]t + 2a_2 t^2 \quad (B)
\end{aligned}
$$

Comparing coefficients in (A) and (B)

$$
\begin{aligned}
2a_2 + 3a_1 + 2a_0 &= 1 \\
6a_2 + 2a_1 &= 2 \\
2a_2 &= 1
\end{aligned}
$$

Solving gives $a_2 = \frac{1}{2}$, $a_1 = -\frac{1}{2}$ and $a_0 = \frac{3}{4}$