Question

Find equations for the following lines in vector and scalar parametric forms, and in standard form:

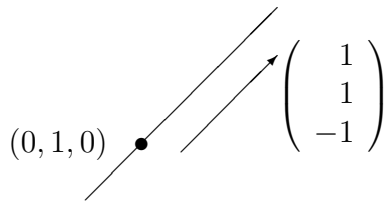
(a) the line that passes through the point $(0, 1, 0)$ and is parallel to $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$;

(b) the line that passes through the points $(1, 2, 2)$ and $(3, -1, 3)$.

Answer

(a) Equation of line in vector parametric form:

$$\begin{pmatrix} x \\ y \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$



so that $x = t,\ y = 1 + t,\ z = -t$ and equation of line in scalar parametric form is: $(x, y, z) = (t, 1 + t, -t)$.

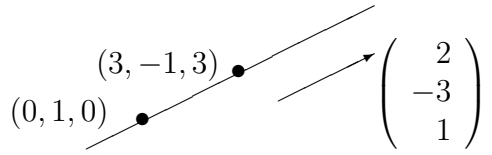
Now,

$$\left.\begin{array}{lll} x = t & \Rightarrow & t = x \\ y = 1 + t & \Rightarrow & t = y - 1 \\ z = -t & \Rightarrow & t = -z \end{array}\right\}.$$

So the equation in standard form is: $x = y - 1 = -z$

(b) The line passes through the point $(1, 2, 2)$ and is parallel to the vector:

$$\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$



Equation in vector parametric form:

$$\begin{pmatrix} x \\ y \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

so that $x = 1 + 2t,\ y = 2 - 3t,\ z = 2 + t$ and equation in scalar parametric form: $(x, y, z) = (1 + 2t, 2 - 3t, 2 + t)$

$$\left.\begin{array}{lll} x = 1 + 2t & \Rightarrow & t = \dfrac{x - 1}{2} \\ y = 2 - 3t & \Rightarrow & t = \dfrac{2 - y}{3} \\ z = 2 + t & \Rightarrow & t = z - 2 \end{array}\right\}$$

So the equation in standard form:

$$\frac{x - 1}{2} = \frac{2 - y}{3} = z - 2$$