Vector Functions and Curves
One variable functions

Question

It is given that the position and velocity vectors of a moving object satisfy $\underline{v}(t) = 2\underline{r}(t)$ for all times t. If $\underline{r}(0) = \underline{r}_0$, find $\underline{r}(t)$ and $\underline{a}(t)$, the acceleration. Also determine the path of motion.

Answer

$$
\begin{aligned}
\frac{d\underline{r}}{dt} &= \underline{v}(t) = 2\underline{r}(t) \\
\underline{r}(0) &= \underline{r}_0
\end{aligned}
$$

And so,

$$
\begin{aligned}
\underline{r}(t) &= \underline{r}(0)e^{2t} = \underline{r}_0 e^{2t} \\
\underline{a}(t) &= \frac{d\underline{v}}{dt} = 2\frac{d\underline{r}}{dt} \\
&= 4\underline{r}_0 e^{2t}
\end{aligned}
$$

The path is a half-line from the origin in the direction of $\underline{r}_0$.