Vector Functions and Curves
Applications

Question

A satellite in a low (radius of orbit is approximately the radius of the Earth) circular orbit passes over both poles. It takes the satellite two hours to make one revolution.

If an observer stands on the equator, what would they make the approximate value of the Coriolis force on the satellite as it passes overhead, heading south?

What direction would the satellite seem to moving to the observer as it passes overhead?

Answer

Assume that the observer is standing at the origin , with $\underline{i}$ pointed to the east and $\underline{j}$ pointed to the north.

The Earth has angular velocity $2\pi/24$ radians per hour northward:

$$\underline{\Omega} = \frac{\pi}{12}\underline{j}.$$

As the Earth is rotating west to east, the actual north to south velocity of the satellite will appear to be moved to the west by $\pi R/12$ km/h, given that R is the radius of the Earth (in km).

As the satellite circles the Earth at a rate of π radians/h, the observer at the origin would take its velocity to be

$$\underline{v}_R = -\pi R\underline{j} - \frac{\pi R}{12}\underline{i}.$$

With $\underline{v}_R$ making an angle with the southward direction of

$$\tan^{-1}\left(\frac{\pi R/12}{\pi R}\right) = \tan^{-1}(1/12) \approx 4.76^{circ}$$

Therefore the satellite appears to be moving a direction which is 4.67° west of south.

And so the apparent Coriolis force is

$$\begin{aligned} -2\underline{\Omega} \times \underline{v}_R &= -\frac{2\pi}{12}\underline{j} \times \left(-\pi R\underline{j} - \frac{\pi R}{12}\underline{i}\right) \\ &= -\frac{\pi^2 R}{72}\underline{k} \end{aligned}$$

This points towards the ground.