QUESTION

(a) A car rental company purchases replacement tyres from a supplier. Usage for the tyres is steady, and the annual usage is 2000 tyres. Storage costs are £20 per tyre per annum.

The tyres can either be purchased from a small local supplier or from a larger national supplier. For the local supplier, the cost of placing an order is £50, and each tyre costs £28.50. For the national supplier, the cost of placing an order is £200 (which reflects the non-local transportation cost), and the purchase price depends on the order size Q as follows:

$$\text{Cost per tyre} = \begin{cases} £28 & \text{for } Q < 300 \\ £27 & \text{for } 300 \leq Q < 600 \\ £26 & \text{for } Q \geq 600. \end{cases}$$

By analysing the relevant costs, determine whether the local or the national supplier should be used, and what order size should be chosen.

(b) Consider the planning of production over four time periods for which the demand is given in the following table.

Period	1	2	3	4
Demand	40	50	20	40

The initial stock level is zero. A set-up cost of £120 is incurred for each period in which there is production. The stock holding cost is £2 per unit of stock held at the end of each period. The objective is to schedule production so that demand is met at minimum cost.

Formulate the problem of finding the minimum cost as a shortest path problem in a network, and hence determine an optimal production schedule.

Find by how much the set-up cost can increase and decrease, without producing a change in the optimal production schedule.

ANSWER

(a) The annual cost (under the usual notation) is

$$K = \frac{sd}{Q} + \frac{1}{2}hQ + cd$$

$\frac{dK}{dQ} = 0$ gives $-\frac{sd}{Q^2} + \frac{1}{2}h = 0$ (if c independent of Q), so $Q = \sqrt{\frac{2sd}{h}}$

- For the local supplier, $Q = \sqrt{\frac{100.2000}{20}} = 100$

$$K = \frac{50.2000}{100} + \frac{1}{2}20.100 + 28.5.2000 = £59000$$

- For the national supplier, $Q = \sqrt{\frac{400.2000}{20}} = 200$
  For $Q = 200$,

$$K = \frac{200.2000}{200} + \frac{1}{2}20.200 + 28.2000 = £60000$$

  For $Q = 300$,

$$K = \frac{200.2000}{300} + \frac{1}{2}.20.300 + 27.2000 = £58333.33$$

  For $Q = 600$

$$K = \frac{200.2000}{600} + \frac{1}{2}.20.600 + 26.2000 = £58666.67$$

The minimum cost is £58333.33, which is for the national supplier with an order size of 300.

(b) If s denotes the set up cost, the network is as follows

DIAGRAM

For $s = 120$, the shortest path length is 400, which is given by path 0 - 1 - 3 - 4. The optimal production schedule is

Period	1	2	3	4
Production	40	70	0	40

For this solution to be optimal, the label at nodes 3 and 4 must be $2s + 40$ and $3s + 40$, respectively. This gives

$s + 40 \leq 25$	$s \geq 40$
$s + 40 \leq 180$	$s \leq 180$
$2s + 40 \leq 3s$	$s \geq 40$
$2s + 40 \leq s + 180$	$s \leq 140$
$2s + 40 \leq 420$	$s \leq 190$

Thus, the required range is $40 \leq s \leq 140$.