QUESTION

An engineering company manufactures industrial machines for overseas export. Demand for the next three months, which must be satisfied, is shown in the following table.

Month	Aug	Sept	Oct
Demand	130	210	220

Capacity restrictions limit the production of machines to a maximum of 200 per month. However, it is possible to manufacture machines before they are required. In this case, there is a cost of £200 per machine in storage at the end of each month. At the start of August, there are no machines in storage, and no machines are required at the end of October.

The machines can be distributed to customers by aeroplane or by ship. Air transportation costs £1000 per machine; sea transportation costs £750 per machine. Machines to be transported by air are dispatched in the month that they are required. On the other hand, machines to be transported by sea must be dispatched one month in advance. As a consequence, all machines that are manufactured to satisfy the demand in August must be transported by air, and no machines are transported by sea in October.

Show that the problem of planning the production and the distribution of machines to customers so that demand is satisfied at minimum total cost can be formulated as a minimum cost network flow problem.

Starting with a solution in which the production quantities, the numbers of machines transported by air and sea, and the end of month inventories, for each month are specified in the following table, use the network simplex method to solve the problem.

Month	Aug	Sept	Oct
Production	160	200	200
Number transported by air	130	180	200
Number transported by sea	30	20	0
End of month inventory	0	0	0

ANSWER

Number the months 1,2,3 and define variables

x_t= production in month t

y_t= quantity transported by air in month t

z_t= quantity transported by sea in month t

s_t= end of month inventory in month t.

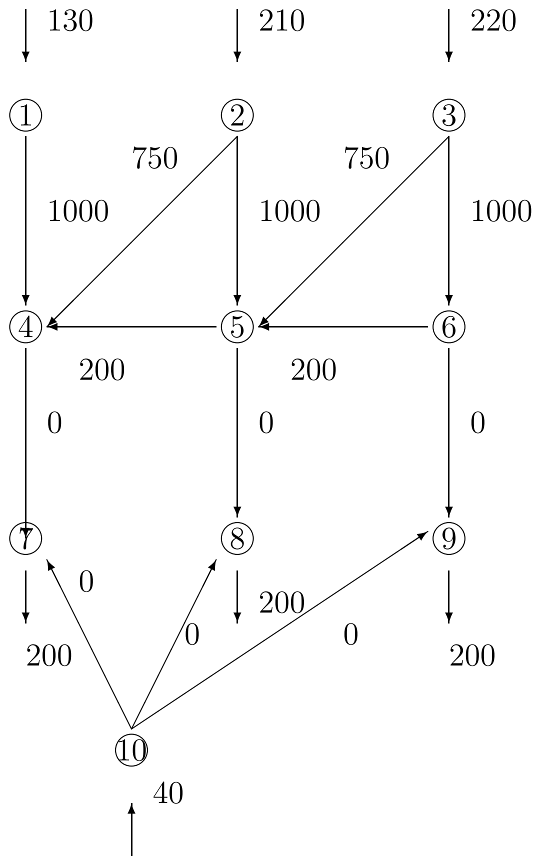
The formulation is

$$
\begin{array}{ll}
\text{Minimize} & z = 1000(y_1 + y_2 + y_3) + 750(z_1 + z_2) + 200(s_1 + s_2) \\
\text{subject to} & x_t \geq 0, y_t \geq 0 \ t = 1, 2, 3 \\
& z_t \geq 0, s_t \geq 0 \ t = 1, 2 \\
& y_1 = 130 \\
& y_2 + z_1 = 210 \\
& y_3 + z_2 = 220 \\
& x_1 - y_x - z_1 - s_1 = 0 \\
& x_2 - y_2 - z_2 + s_1 - s_2 = 0 \\
& x_3 - y_3 - z_3 + s_2 = 0 \\
& x_1 \leq 200 \\
& x_2 \leq 200 \\
& x_3 \leq 200
\end{array}
$$

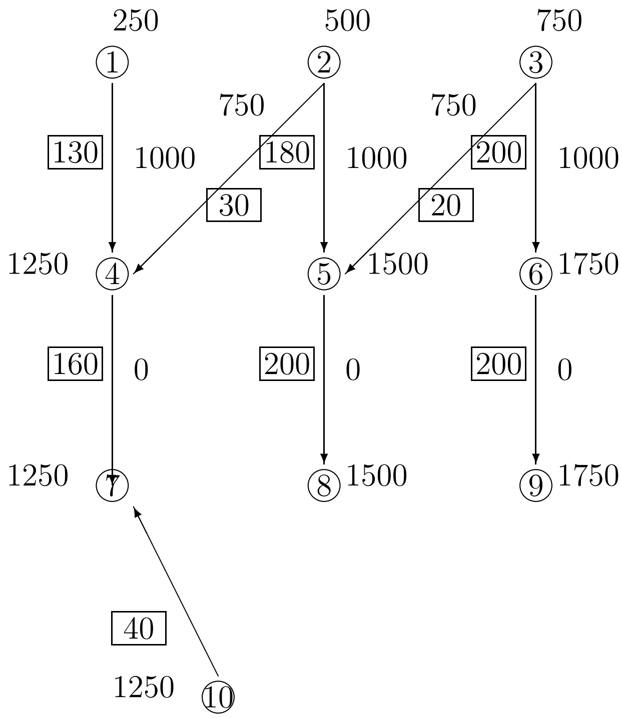
The last 3 constraints can be written as

$$
\begin{aligned}
-x_1 - t_1 &= -200 \\
-x_2 - t_2 &= -200 \\
-x_3 - t_3 &= -200
\end{aligned}
$$

for slack variables t_1, t_2, t_3, t_4 and a redundant constraint is $t_1 + t_2 + t_3 = 40$.

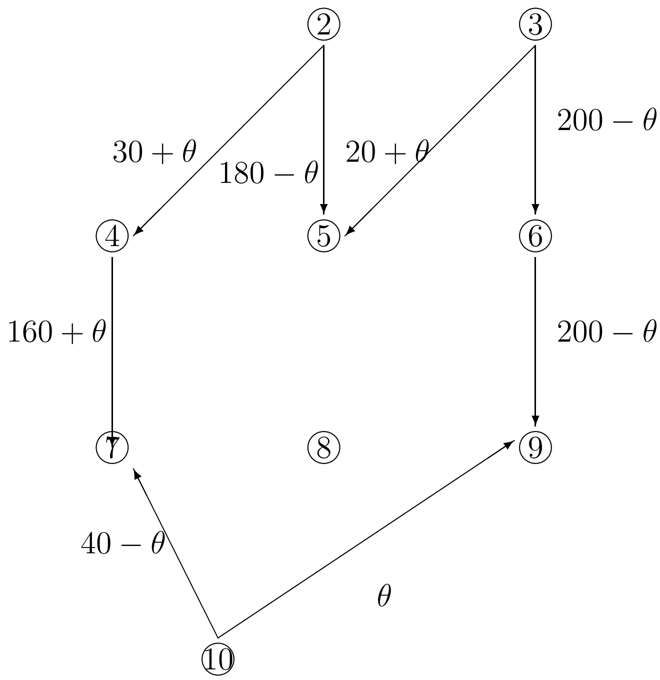


The initial tree solution is

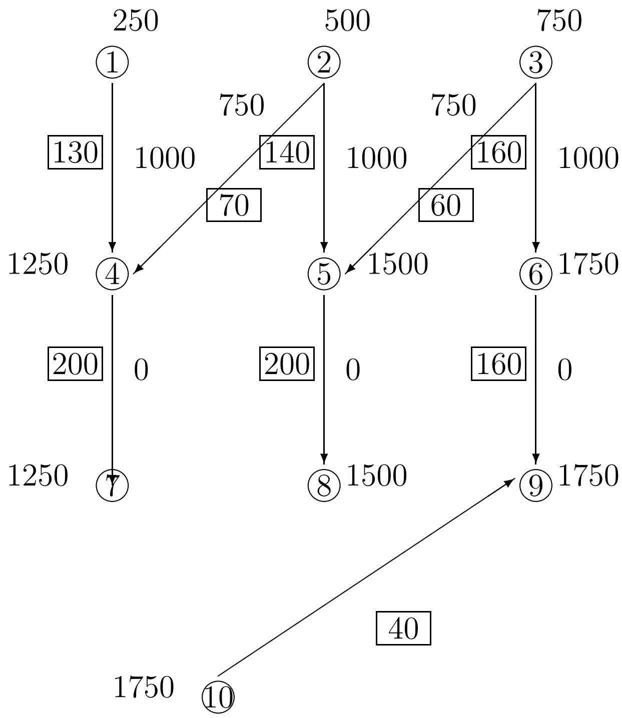


(i,j)	$y_i + c_{ij} - y)j$
(5,4)	450
(6,5)	450
(10,8)	−250
(10,9)	−500

Entering are (10,9)



$\theta = 40$



(i,j)	$y_i + c_{ij} - y_j$
(5,4)	450
(6,5)	450
(10,7)	500
(10,8)	250

Optimal solution is

	Aug	Sept	Oct
Production	200	200	160
Trans by air	130	140	160
Trans by sea	70	60	0

$$z = 430 \times 1000 + 130 \times 750 = 527500$$