QUESTION

Decide for each of the following statements whether or not it is true giving a brief explanation of your answer.

(i) For each positive integer $n \geq 2$ the symmetric group S_n has a subgroup of index 2.

(ii) The function $f : D_n \longrightarrow Z_2$ defined by $f(g) = 1$ if and only if g is a rotation (The set of rotations includes the identity) and $f(g) = 0$ if and only if g is a reflection is a homomorphism.

(iii) There are precisely 48 elements in the cyclic group Z_{180} with the property that they each generate the whole group.

(iv) Given any finite group G there is a positive integer n such that G is isomorphic to a subgroup of S_n.

(v) Every group of even order is abelian.

(vi) If G is a finite group of order n then $g^n = e$ for every element $g \in G$.

ANSWER

(i) True, $A_n < S_n$ has index 2

(ii) False, $f(\rho^2) = 1$ but $f(\rho) + f(\rho) = 0$

(iii) True, Number of generators of Z_{180} is $\phi(18))$. $180 = 2^2.5.9$ so $\phi(180) = \phi(2^2).\phi(5).\phi(9) = 1.4.6 = 48$

(iv) True, Cayley's theorem gives an isomorphism from G to a subgroup of S_g and thus into $S_{|G|}$.

(v) False, D_3 is not abelian but has order 6

(vi) True, By Lagrange's theorem the order d of g divides n so $g^n = (g^d)^{\frac{n}{d}} = e$.