Exam Question
Topic: Laplace
Write down the Laplace transforms of the following functions

$$\text{(i)} \quad \sin 3t + t^2 \qquad \text{(ii)} \quad \left(1 + \mathrm{e}^{-t}\right)^2 \qquad \text{(iii)} \quad t\cosh t.$$

Find the inverse transform of

$$\frac{6p^2\mathrm{e}^{-p}}{(p^2-1)(p^2+2)}$$

Solution

(i) $L(\sin 3t + t^2) = \dfrac{3}{p^2+9} + \dfrac{2}{p^3}.$

(ii) $L\left((1+\mathrm{e}^{-t})^2\right) = L(1 + 2\mathrm{e}^{-t} + \mathrm{e}^{-2t}) = \dfrac{1}{p} + \dfrac{2}{p+1} + \dfrac{1}{p+2}.$

(iii) $L(t\cosh t) = L\left(\dfrac{t\,\mathrm{e}^{t} + t\,\mathrm{e}^{-t}}{2}\right) = \dfrac{1}{2(p-1)^2} + \dfrac{1}{p+2}.$

$$\frac{6p^2\mathrm{e}^{-p}}{(p^2-1)(p^2+2)} = \mathrm{e}^{-p}\left(\frac{2}{p^2-1} + \frac{4}{p^2+2}\right) = \left(\frac{1}{p-1} + \frac{1}{p+1} + \frac{4}{p^2+2}\right)\mathrm{e}^{-p}$$

Inverting using the second shift theorem gives

$$L^{-1}\left(\frac{6p^2\mathrm{e}^{-p}}{(p^2-1)(p^2+2)}\right) = \left(\mathrm{e}^{(t-1)} - \mathrm{e}^{-(t-1)} + \frac{4}{\sqrt{2}}\sin\left(\sqrt{2}(t-1)\right)\right)H(t-1).$$