QUESTION

(i) Find HCF(1147,851).

(ii) Find HCF(148,1147,851)

(iii) Find all the integral solutions, x and y, to the linear Diophantine equation

$$1147x + 851y = 111.$$

ANSWER

(i) We use the Euclidean algorithm

$$
\begin{aligned}
1147 &= 1 \times 851 + 296 \\
851 &= 2 \times 296 + 259 \\
296 &= 1 \times 259 + 37 \\
269 = 7 \times 37
\end{aligned}
$$

So that HCF(1147, 851)=37.

(ii) By part (i), HCF(148,1147,851)=HCF(148,37)=37 since $148 = 4 \times 37$ and 37 is a prime.

(iii) To solve this we must first observe that $111 = 3 \times 37$ so that there exists an infinite number of solutions. Next we must find one.

From the Euclidean algorithm in (i), $296 = 1147 - 1 \times 851$ and $259 = 1 \times 851 - 2 \times (11 - 1 \times 851) = 3 \times 851 - 2 \times 1147$ so that

$$37 = 1147 - 1 \times 851 - (3 \times 851 - 2 \times 1147) = 3 \times 1147 - 4 \times 851.$$

Therefore

$$11 = 9 \times 1147 - 12 \times 851$$

so that one solution is $x = 9, y = -12$ and therefore the general solution is

$$x = 9 + \left(\frac{851n}{37}\right), y = -12 - \left(\frac{1147n}{37}\right)$$

where n is an arbitrary integer.

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