QUESTION

Find all the solutions of each of the following congruences, expressing your answers in terms of congruence classes, $[x]$:

(i) $7x \equiv 29 \bmod (51)$,

(ii) $6(x^5 + x^3 + 3001) \equiv 23 \bmod (48)$,

(iii) $x^2 - x + 2 \equiv 0 \bmod (4)$.

ANSWER

(i) Since HCF(7,51)=1 there is exactly one congruence class $[x]$ of solutions. We can find this either by using the Euclidean algorithm to find u and v such that $7u+51v = 1$ then $[x] = [29u]$ or we can find the solution as follows. The equation is the same mod (51) as $-44x \equiv -22 \bmod (51)$ which has the same solutions as $2x \equiv 1 \bmod (510$. The solution to this is $[x] = [26]$, which is correct since $26\times 7 = 182 = 153+29 = 3\times 51+29$, as required.

(ii) Since HCF(6,48)=6 does not divide 23 there are no solutions. Put another way, if $6(x^5 + x^3 + 3001) = 23 + 48n$ then taking remainder mod (6) on both sides gives the contradiction that $23 \equiv 0 \bmod (6)$.

(iii) It suffices to try substituting the values $[x] = [0], [1], [2], [3]$ since the congruence class mod (4) of the left hand side depends only on $[x]$. Here is a table:

$[x]$	$[x^2]$	$[x]$	$[x^2 - x + 2]$
$[0]$	$[0]$	$[0]$	$[2]$
$[1]$	$[1]$	$[1]$	$[2]$
$[2]$	$[0]$	$[2]$	$[0]$
$[3]$	$[1]$	$[3]$	$[0]$

so the solutions are all integers $x \equiv 2, 3 \bmod (4)$.