QUESTION Use the Cauchy-Riemann equations to show that the function $f(z) = xy + iy$ is nowhere analytic.

ANSWER By the Cauchy-Riemann equations this function can only be differentiable at the origin and nowhere else. Thus it is not differentiable in any region, and so analytic nowhere.