

Question

Two fair dice are thrown. What is the probability of obtaining:

- (a) A total of exactly 7?
- (b) A total of 7 or more?
- (c) A total of 7 or less?

Answer

- (a) To obtain 7 exactly, one needs the following pairs

$\{1,6\}, \{6,1\}, \{2,5\}, \{5,2\}, \{3,4\}, \{4,3\}$

each possibility has a probability $\frac{1}{36} = \left(\frac{1}{6}\right) \times \left(\frac{1}{6}\right)$

There are six possibilities, six elements in the sample space

Hence $p(a_1 + a_2 = 7) = \frac{6}{36} = \frac{1}{6}$

- (b)

To obtain 12 need $\{6,6\}$ $p = \frac{1}{36}$

11 need $\{6,5\}, \{5,6\}$ $p = \frac{1}{18}$

10 need $\{6,4\}, \{4,6\}, \{5,5\}$ $p = \frac{1}{12}$

9 need $\{6,3\}, \{3,6\}, \{4,5\}, \{5,4\}$ $p = \frac{4}{36} = \frac{1}{9}$

8 need $\{6,2\}, \{2,6\}, \{3,5\}, \{5,3\}, \{4,4\}$ $p = \frac{5}{36}$

Hence $p(a_1 + a_2 \geq 7) = \frac{1}{36}(1 + 2 + 3 + 4 + 5 + 6) = \frac{21}{36} = \frac{7}{12}$

- (c)

$$\begin{aligned} p(a_1 + a_2 \leq 7) &= 1 - p(a_1 + a_2 > 7) \\ &= 1 - p(a_1 + a_2 \geq 7) + p(a_1 + a_2 = 7) \\ &= 1 - \frac{7}{12} + \frac{1}{6} \\ &= \frac{7}{12} \end{aligned}$$