QUESTION

(a) Express -16 in exponential form and hence find all the complex values of $(-16)^{\frac{1}{4}}$, writing your answers in the form $x + jy$.

Display these values on an Argand diagram.

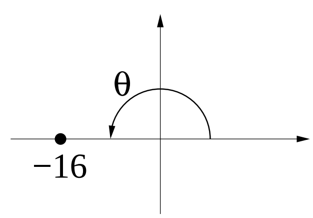
(b) Using Laplace transforms find, with the aid of tables, the solution of the ordinary differential equation

$$\frac{dx}{dt} + 2x = 1$$

which satisfies the condition $x = 2$ when $t = 0$.

ANSWER

(a)



$\theta = \pi$ therefore $-16 = 16e^{j\pi} = 16e^{j(\pi+2k\pi)}$ $k = 0, \pm1, \pm2, \ldots$

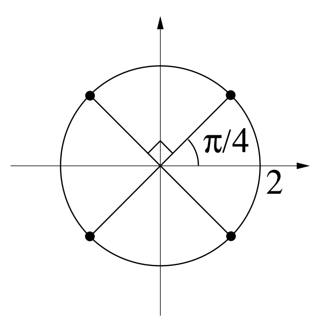
so $(-16)^{\frac{1}{4}} = (16)^{\frac{1}{4}}e^{j\frac{(\pi+2k\pi)}{4}}$

$k = 0,\ 2e^{j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \sqrt{2}(1+j)$

$k = 1,\ 2e^{j\frac{3\pi}{4}} = 2\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right) = 2\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \sqrt{2}(-1+j)$

$k = 2,\ 2e^{j\frac{5\pi}{4}} = 2\left(\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right) = 2\left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = \sqrt{2}(-1-j)$

$k = 3,\ 2e^{j\frac{7\pi}{4}} = 2\left(\cos\frac{7\pi}{4} + j\sin\frac{7\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = \sqrt{2}(1-j)$



(b) $\frac{dx}{dt} + 2x = 1,\ x = 2$ when $t = 0$

Taking Laplace transforms $\mathcal{L}\left\{\frac{dx}{dt} + 2x\right\} = \mathcal{L}\left\{\frac{dx}{dt}\right\} + 2\mathcal{L}\{x\} = \mathcal{L}\{1\}$

$\{sX - x(0)\} + 2X = \frac{1}{s}$

$(s+2)X = \frac{1}{s} + 2,\ X = \frac{1}{(s+2)s} + \frac{2}{s+2}$

$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} = \frac{A(s+2) + Bs}{s(s+2)}$

$1 = A(s+2) + Bs$

$s = 0,\ 1 = 2A,\ A = \frac{1}{2}$

$s = -2,\ 1 = -2B,\ B = -\frac{1}{2}$

Therefore $X = \frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}}{s+2} + \frac{2}{s+2} = \frac{\frac{1}{2}}{s} + \frac{\frac{3}{2}}{s+2}$

Taking the inverse Laplace transform gives $x(t) = \frac{1}{2} + \frac{3}{2}e^{-2t}$.