QUESTION

The point $A(2,3,1)$, $B(0,1,2)$ and $C(2,-1,-1)$ lie on a plane.

(i) Write down the vectors $\vec{AB}$ and $\vec{AC}$.

(ii) Obtain a unit vector perpendicular to the plane.

(iii) Derive the vector equation of the plane.

(iv) Find the perpendicular distance from the point $D(5,2,2)$ to the plane.

(v) Obtain the coordinates of the point at which the perpendicular from D to the plane intersects the plane.

ANSWER

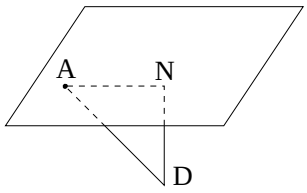
$A(2,3,1), \qquad B(0,1,2), \qquad C(2,-1,-1)$

(i) $\vec{AB} = (0-2, 1-3, 2-1) = (-2,-2,1)$
$\vec{AC} = (2-2, -1-3, -1-1) = (0,-4,-2)$

(ii) $\vec{AB} \times \vec{AC} = (-2,-2,1) \times (0,-4,-2) = (4-(-4), 0-4, 8-0) = (8,-4,8)$.
To find the unit vector divide by the magnitude.
Now $|\vec{AB} \times \vec{AC}| = (8^2 + (-4)^2 + 8^2)^{\frac{1}{2}} = (64+16+64)^{\frac{1}{2}} = 12$,
therefore the unit vector is $\dfrac{1}{12}(8,-4,8) = \left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{2}{3}\right)$

(iii) The vector equation of the plane is $\mathbf{r.n} = c = \mathbf{a.n}$
Therefore $\mathbf{r.n} = (2,3,1).\left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{2}{3}\right) = \dfrac{4}{3} - 1 + \dfrac{2}{3} = 1$
i.e. $\mathbf{r}.\left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{2}{3}\right) = 1$ or $\mathbf{r}.(2,-1,2) = 3$

(iv)



Distance from D to plane $= \left|\vec{DA}.\hat{\mathbf{n}}\right| = \left|(-3,1,-1).\left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{2}{3}\right)\right| = \left|-2 - \dfrac{1}{3} - \dfrac{2}{3}\right| = |-3| = 3$

(v) Coordinates of N are

$$(5,2,2) - 3\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right) = (5,2,2) - (2,-1,2) = (3,3,0)$$

(Other methods could have been used).