Question

For each of the following equations, determine whether the equation describes a Euclidean line in $\mathbf{C}$ or a circle in $\mathbf{C}$ (or neither). In the former case, give its slope and y-intercept. In the latter case, give its center and radius.

(a) $5z\overline{z} + (3+i)z + (3-i)\overline{z} + 6 = 0$;

(b) $(-2-3i)z + (-2+3i)\overline{z} + 2 = 0$;

(c) $-z\overline{z} - 2iz + 2i\overline{z} + 1 = 0$;

Answer

(a) $5z\bar{z} + (3+i)z + (3-i)\bar{z} + 6 = 0$

$$z\bar{z} + \frac{3+i}{5}z + \frac{3-i}{5}\bar{z} + \frac{6}{5} = 0$$

$$\left(z + \frac{3-i}{5}\right)\left(\bar{z} + \frac{3+i}{5}\right) - \frac{(3+i)(3-i)}{25} + \frac{6}{5} = 0$$

$$\left(z - \frac{(-3+i)}{5}\right)\left(\bar{z} - \frac{(-3-i)}{5}\right) - \frac{10}{25} + \frac{30}{25} = 0$$

$$\left|z - \frac{(-3+i)}{5}\right|^2 = -\frac{20}{25}$$

so no solutions.

(b) $(-2-3i)z + (-2+3i)\bar{z} + 2 = 0$ (euclidean line)

$$(-2-3i)(x+iy) + (-2+3i)(x-iy) + 2 = 0$$

$$-2x + 3y - 2iy - 3ix - 2x + 3y + 3ix + 2iy + 2 = 0$$

$$-4x + 6y + 2 = 0$$

$$\begin{aligned} 6y &= 4x - 2 \\ y &= \frac{2}{3}x - \frac{1}{3} \end{aligned}$$

slope $\frac{2}{3}$

y-intercept $-\frac{1}{3}$

(c) $z\bar{z} - 2iz + 2i\bar{z} + 1 = 0$

$z\bar{z} + 2iz - 2i\bar{z} - 1 = 0$

$(z - 2i)(\bar{z} + 2i) - 4 - 1 = 0$

$|z - 2i|^2 = 5$

$|z - 2i| = \sqrt{5}$

euclidean circle center $2i$, radius $\sqrt{5}$.