

Question

Sketch or describe the level sets for each of the following functions $f : \mathbb{R}^3 \rightarrow \mathbb{R}$

:

$$f(x_1, x_2, x_3) = \begin{array}{ll} \text{(a)} & x_1^2 + 2x_2^2 + 3x_3^2 \\ \text{(b)} & x_1^2 - 2x_2^2 + 3x_3^2 \\ \text{(c)} & x_1^2 + x_2^2 \end{array} \quad \begin{array}{ll} \text{(d)} & x_1^2 \\ \text{(e)} & x_1^2 + x_2^2 + x_3 \\ \text{(f)} & x_1^2 + x_3 \end{array}$$

Answer

(a) Ellipsoids

(b) $c = 0$: cone, axis = y-axis

$c > 0$: hyperboloid of 1 sheet

$c < 0$: hyperboloid of 2 sheets

(c) Cylinders, axis = x-axis

(d) Plane pairs $x = \pm\sqrt{c}$ ($c > 0$)

(y, z) plane $x = 0$ ($c = 0$)

empty when $c < 0$.

(e) Paraboloids with axis = x-axis (put $x^2 + y^2 = r^2$ and compare (d))

(f) Parabolic 'troughs': in (d) above, replace y by x and then slide the whole picture along the third (=new y-) axis.