Question
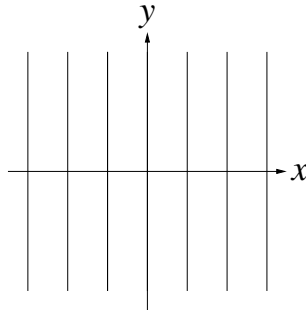
Sketch the contours for each of the following functions $f : \mathbf{R}^2 \to \mathbf{R}$:

$$f(x_1, x_2) = \quad \begin{array}{ll} \textbf{(a)} & x_1^2 \\ \textbf{(b)} & x_1^2 - 2x_1x_2 + x_2^2 \\ \textbf{(c)} & x_1^2 - 2x_1x_2 + 2x_2^2 \end{array} \quad \begin{array}{ll} \textbf{(d)} & x_1^2 + x_2 \\ \textbf{(e)} & x_1^2 - 2x_1x_2 \\ \textbf{(f)} & \sin x_1 \sin x_2 \end{array}$$
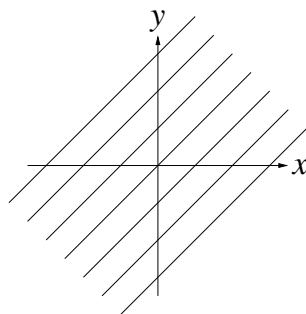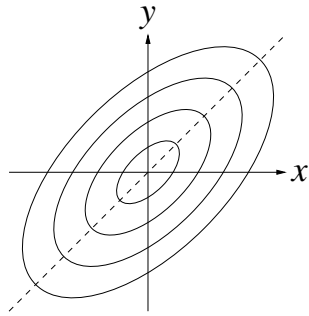
Answer

(a)

$$x^2 = c, \quad \begin{aligned} x &= \pm\sqrt{c} \quad (c > 0) \\ &= 0 \quad (c = 0) \end{aligned}$$

(b)

$$x^2 - 2xy + y^2 = (x - y)^2 = c$$
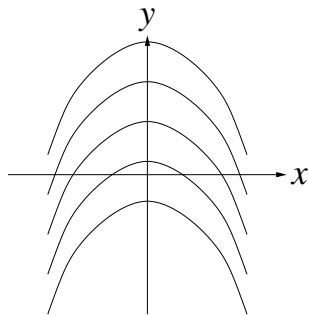
(c)

Ellipses

$$
\begin{aligned}
x - 2xy + 2y^2 &= (x-y)^2 + y^2 \\
&= u^2 + v^2 = c
\end{aligned}
$$

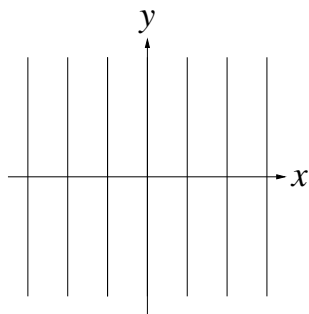**In co-ordinates $x = u + v$, $y = v$.**

(d)

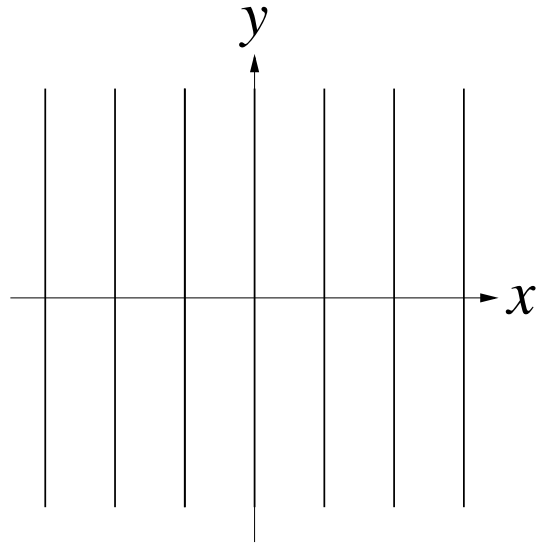**Parabolas**

$$y = -x^2 + c$$

(e)

$$x^2 - 2xy = x(x - 2y) = c$$

$c = 0$: **Pair of lines**

$c \neq 0$: **Hyperbolae**

(f)

$c = 0$: **lines** $x = n\pi$, $y = m\pi$ $(m, n \in \mathbf{Z})$.

$c = 1$: **max points** *

$c = -1$: **min points** ●

empty when $|c| > 1$