Question
What can be said about a sequence $\{a_n\}$ if it converges and if every a_n is an integer? Also, give a qualitative description of all of the convergent subsequences of the sequence

$$1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, \ldots.$$

Answer
A convergent sequence of integers must be eventually constant; that is, there exists M so that $a_n = a_p$ for all n, $p > M$. This follows from the Cauchy criterion with $\varepsilon = \frac{1}{2}$ and the fact that the difference of two non-equal integers is at least 1.

For this given sequence, the convergent subsequences are all of the following form: pick a positive integer p, and note that p appears infinitely many times in the given sequence. Then, a convergent subsequence is of the form $a_0, a_1, \ldots, a_M, a_{M+1} = p, a_{M+2} = p, \ldots$ for some M, where $a_0, \ldots, a_M$ are arbitrary positive integers.