Question
By explicitly calculating its partial sums, show that the infinite series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

is convergent.

Answer
Calculating, we see that the k^{th} partial sum is a telescoping sum, namely

$$S_k = \sum_{n=1}^{k} \left(\frac{1}{n} - \frac{1}{n+1} \right) = \left(\frac{1}{1} - \frac{1}{1+1} \right) + \left(\frac{1}{2} - \frac{1}{2+1} \right) + \cdots + \left(\frac{1}{k} - \frac{1}{k+1} \right) = 1 - \frac{1}{k+1}.$$

Therefore, $\lim_{k\to\infty} S_k = 1 - \lim_{k\to\infty} \frac{1}{k+1} = 1$, and so this series converges.