Question
State both parts of the Fundamental Theorem of Calculus. Also, determine whether the following argument is correct: By the Fundamental Theorem of Calculus,

$$\int_{-1}^{1} \frac{1}{x^2} \mathrm{d}x = \left[-\frac{1}{x}\right]_{-1}^{1} = -2,$$

and so the integral of a positive function can be negative.

Answer
Fundamental theorem of calculus: Let f be a continuous function on the closed interval $[a, b]$.

- Consider the function on $[a, b]$ defined by

  $$F(x) = \int_a^x f(t) \mathrm{d}t.$$

  Then, $F'(x) = f(x)$ for every x in (a, b). In shorthand,

  $$f(x) = \frac{\mathrm{d}}{\mathrm{d}x} \int_a^x f(t) \mathrm{d}t.$$

- If G is any function on $[a, b]$ satisfying $G'(x) = f(x)$, then

  $$\int_a^b f(x) \mathrm{d}x = G(b) - G(a).$$

  In shorthand,

  $$\int_a^b G'(x) \mathrm{d}x = G(b) - G(a).$$

The proof is false: the integrad is not continuous on $[-1, 1]$, and so the fundamental theorem of calculus does not apply.