Question
Determine whether the improper integral

$$\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$$

converges or diverges. In the case that it converges, determine its value.

Answer

$$
\begin{aligned}
\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x &= \lim_{\varepsilon \to 0+} \int_\varepsilon^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x \\
&= \lim_{\varepsilon \to 0+} 2e^{\sqrt{x}}\Big|_\varepsilon^1 \\
&= \lim_{\varepsilon \to 0+} \left(2e - 2^{\sqrt{\varepsilon}}\right) = 2e - 2,
\end{aligned}
$$

which converges.