Question

Show that the function $h(x) = \sqrt{x-1}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[2, 5]$. Find all the numbers c in $(2, 5)$ that satisfy the conclusion of the Mean Value Theorem.

Answer

(Be sure to state the mean value theorem first, so that it is clear that you know what the hypotheses and the conclusions are.) Note that $h(x)$ is continuous and differentiable on all of $(1, \infty)$, since $x - 1 > 0$ on $x > 1$, and so in particular h is continuous on $[2, 5]$ and differentiable on $(2, 5)$ (i.e., satisfies the hypotheses).

So, there exists some c in $(2, 5)$ at which

$$h'(c) = \frac{h(5) - h(2)}{5 - 2} = \frac{1}{3}.$$

In fact, since $h'(c) = \frac{1}{2\sqrt{c-1}}$, the only solution to $h'(c) = \frac{1}{3}$ occurs at $c = \frac{13}{4}$ (which does lie in $(2, 5)$, as expected).