Question

Let f be a function which is continuous on the closed interval $[a, b]$, where $a < b$. Suppose that $f(b) < f(a)$. Determine whether there exists a point c in the open interval (a, b) so that $f(c) = c$.

Answer

Not necessarily: take $f(x) = 100 - x$ on the interval $[a, b] = [0, 1]$. Then, $f(1) = 99 < f(0) = 100$, but there are no solutions to $x = 100 - x$ in the interval $[0, 1]$. (The only solution is at $x = 50$.)