

Question

In analysing the linear-free vibrations of a uniform clamped-beam, one encounters the eigenvalue problem

$$\cos \lambda \cosh \lambda = -1$$

Show that for n integer,

$$\lambda \sin \left(n - \frac{1}{2} \right) \pi + 2e^{-(n-\frac{1}{2})\pi} \sin \left(n - \frac{1}{2} \right) \pi, \quad n \rightarrow +\infty$$

Answer

$\cosh \lambda \rightarrow \infty$ as $\lambda \rightarrow \infty$ so

$$\cos \lambda = -\frac{1}{\cosh \lambda} \rightarrow 0 \text{ as } \lambda \rightarrow \infty$$

Therefore we look for large λ solutions $\lambda \approx \left(n - \frac{1}{2} \right) \pi$ n integer, being the zeros of $\cos \lambda$.

So try $\lambda = \left(n - \frac{1}{2} \right) \pi + \delta$, $\delta = o(1)$.

Substitute back into full equation.

$$\cos \left(\left(n - \frac{1}{2} \right) \pi + \delta \right) = -\frac{1}{\cosh \left(\left(n - \frac{1}{2} \right) \pi + \delta \right)}$$