

QUESTION

Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{z^n}{\sqrt{n}}.$$

Find one point on the circle of convergence where this series converges and one point on the circle of convergence where it diverges.

ANSWER

$R = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+1}}$ . At  $z = 1$  we have  $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots$  which diverges by the comparison test. At  $z = -1$  the series converges by the alternating series test.