QUESTION

For which values of z is $\tanh z$ *not* analytic? What is the largest circle (centre at tha origin) for which the Taylor series about $z = 0$ for $\tanh z$ converges to $\tanh z$?. Find the first two non-zero terms ofthis series.

ANSWER

$\tanh z = \sinh z / \cosh z$ is not analytic if and only if $\cosh z = 0$. This is when $e^z + e^- z = 0$ or when $e^{2z} = -1 = e^{i\pi}$. Thus a point closest to the origin where we do not get convergence is $z = i\pi/2$, so we get convergence if $|z| < \pi/2$. We compute the Taylor series in usual way as in question 1.