QUESTION

Find the Taylor series for $\cos z$ about $z = \pi/2$.

ANSWER

Put
$$w = z - \pi/2$$
. Then $\cos z = \cos(w + \pi/2)$. Put $g(w) = \cos(w + \pi/2) = g(0) + wg'(0) + \frac{w^2}{2!}g''(0) + \cdots$. Now $g(0) = 0, g'(0) = -1 \cdots$ so

$$\cos z = -(z - \pi/2) + (z - \pi/2)^3/3! + \cdots$$