

Question

Suppose that an integer between (including) 1 and 100 is chosen at random and that X denotes the number obtained. Determine the distribution of X , and find the mean and variance of X .

Answer

From the assumption that X has a discrete uniform distribution over $1, 2, \dots, 100$ i.e. $P\{X = k\} = \frac{1}{100}$, $k = 1, 2, \dots, 100$.

$$E(X) = \frac{1}{100} \sum_{k=1}^{100} k = 50.5$$

$$E(X^2) = \frac{1}{100} \sum_{k=1}^{100} k^2 = 3383.5$$

$$\text{var}(X) = E(X^2) - \{E(X)\}^2 = 833.25$$

$$\sum_{i=1}^N i = \frac{N(N+1)}{2}$$

$$\sum_{i=1}^N i^2 = \frac{N(N+1)(N+2)}{6}$$

$$\text{var}(X) = \frac{N^2 - 1}{12} \text{ for discrete uniform}$$