

Question

Calculate the Lebesgue integral $L \int_{[0,1]} f$ where $f : \mathbf{R} \rightarrow \mathbf{R}$.

$$f(x) = \begin{cases} 1 & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

Answer

$$L \int_{[0,1]} f \quad \text{Let } g = 0, \quad f = g \text{ a.e.}$$

$$\text{So } L \int_{[0,1]} f = L \int_{[0,1]} g = R \int_{[0,1]} g = 0$$