Question

If $f : \mathbf{R^n} \rightarrow \mathbf{R}^*$ is integrable, show that, for any $\epsilon > 0$,
$m\{x||f(x)| \geq \epsilon\} < +\infty$
Is it necessarily true that $m(\{x||f(x)| > 0\} < +\infty$?

Answer

$\{x||f(x)| \geq \epsilon\} = \{x|f(x) \geq \epsilon\} \cup \{x|f(x) < -\epsilon\}$
Suppose $m(\{x|f(x)| \geq \epsilon\}) = +\infty$
Then $\int f_+ > \epsilon_x + \infty = +\infty$ and so f is not integrable.
Let $f(x) = e^{-|x|}$. Then $\displaystyle\int e^{-|x|} = 2\int_0^\infty e^{-x} = 2[-e^{-x}]_0^\infty = 2 < \infty$
$\{x|e^{-|x|} > 0\} = \mathbf{R^0} \;\; m\mathbf{R} = +\infty$