QUESTION

Find the adjoints of the following and, if they exist, the inverses:

$$\begin{bmatrix} 2 & -1 & 4 \\ -1 & 0 & 5 \\ 9 & -7 & 3 \end{bmatrix}, \qquad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 2 & 3 \\ -1 & 6 & -2 \\ -1 & 22 & 0 \end{bmatrix}.$$

ANSWER

First matrix:

$$\mathrm{adj}\,A = \begin{bmatrix} 35 & -25 & -5 \\ 48 & -30 & -14 \\ 7 & 5 & -1 \end{bmatrix};$$

$A\,\mathrm{adj}\,A = 50I$; $\det A = 50$; $A^{-1} = \dfrac{\mathrm{adj}\,A}{50}$

Second matrix:

$$\mathrm{adj}\,A = \begin{bmatrix} 2 & -1 & -1 & -1 \\ -1 & 2 & -1 & -1 \\ -1 & -1 & 2 & -1 \\ -1 & -1 & -1 & 2 \end{bmatrix};$$

$A\,\mathrm{adj}\,A = -3I$; $\det A = -3$; $A^{-1} = \dfrac{\mathrm{adj}\,A}{-3}$

Third matrix:

$$\mathrm{adj}\,A = \begin{bmatrix} 44 & 66 & -22 \\ 2 & 3 & -1 \\ -16 & -24 & 8 \end{bmatrix};$$

$A\,\mathrm{adj}\,A = 0$; no inverse.