QUESTION

Show that

$$A = \begin{bmatrix} 5 & 1 & 1 \\ -12 & 12 & 3 \\ -4 & 1 & 10 \end{bmatrix}$$

has a triple eigenvalue 9 but that 9 has only a two dimensional eigenspace ($4x = y + z$) so A cannot be diagonalised. Choose a vector $\mathbf{b}$ not in the eigenspace and let $\mathbf{a} = (A - 9I)\mathbf{b}$

Show that $\mathbf{a}$ is an eigenvector. Choose a second eigenvector $\mathbf{c}$ independent of $\mathbf{a}$ and form the matrix M which has as its columns the vectors **a,b,c**. Calculate M^{-1} and show that the matrix

$$\Lambda = M^{-1}AM = \begin{bmatrix} 9 & 1 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

ANSWER There are many choices for $\mathbf{b}$ but to make the calculation as simple as possible, start with $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Then

$$\mathbf{a} = (A - 9I)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Another simple choice is $\mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and these choices give

$$\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 5 & 1 & 1 \\ -12 & 12 & 3 \\ -4 & 1 & 10 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 & 1 \\ -36 & 9 & 9 \\ -27 & 9 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$