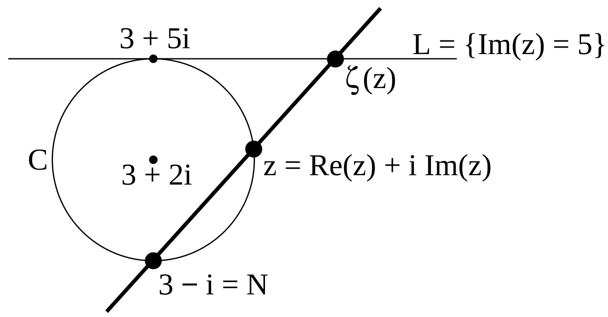
Question

Write down an explicit formula for the stereographic projection map from the Euclidean circle in $\mathbf{C}$ with center $3+2i$ and radius 3 to the horizontal Euclidean line through $3+5i$ (union $\{\infty\}$).

Answer



(remember to project from the point on the circle C opposite from the point of tangency of C and L).
equation of line through $N,\ Z$:

- $m = \dfrac{-1-\mathrm{Im(z)}}{3-\mathrm{Re(z)}}$
- equation:
  $$\begin{aligned} y+1 &= m(x-3) \\ y+1 &= \frac{-1-\mathrm{Im(z)}}{3-\mathrm{Re}(z)}(x-3) \end{aligned}$$
- Set $y=5$ (to get the intersection with L) and solve for x:
  $$-6\cdot\frac{3-\mathrm{Re}(z)}{1+\mathrm{Im(z)}} = x-3$$
  $$x = -6\cdot\frac{3-\mathrm{Re}(z)}{1+\mathrm{Im(z)}} + 3.$$

So $\xi(z) = \begin{cases} -6\cdot\dfrac{3-\mathrm{Re}(z)}{1+\mathrm{Im(z)}} + 3 + 5i & z\neq N \\ \infty & z = N \end{cases}$