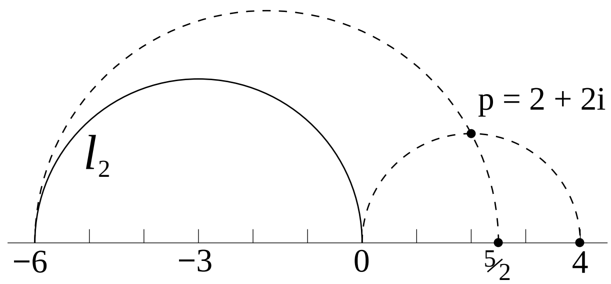
Question
Let ℓ_1 be the hyperbolic line contained in the Euclidean line $\{z \in \mathbf{C} \,|\, \mathrm{Re}(z) = 4\}$, let ℓ_2 be the hyperbolic line contained in the Euclidean circle with center -3 and radius 3, and let p be the point $p = 2 + 2i$. Determine explicitly all the hyperbolic lines through p which are parallel to both ℓ_1 and ℓ_2.

Answer
Let's take it one line at a time:



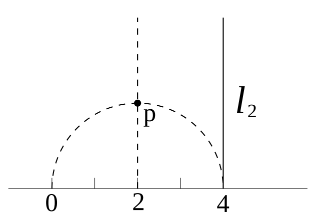
Consider the two lines through p and the two endpoints at infinity of ℓ_2:
The line through p and 0 has endpoints at infinity 0,4.
The line through p and -6 has endpoints at infinity $-6, \frac{5}{2}$ (of center and radius of the euclidean circle containing the hyperbolic line through p, -6).
These 4 points $-6, 0, \frac{5}{2}, 4$ break $\mathbf{R} \cup \{\infty\}$ into 4 intervals:

$$(-6, 0), \ \left[0, \tfrac{5}{2}\right], \ \left(\tfrac{5}{2}, 4\right), \ [4, -6]$$

(where $[4, -6] = [4, \infty] \cup \{\infty\} \cup (-\infty, -6]$ is an interval through ∞).
A hyperbolic circle through p intersects ℓ_2 if and only if it has an end point at infinity in (-6,0) or $\left(\frac{5}{2}, 4\right)$. So, the hyperbolic lines through p and parallel to ℓ_2 correspond to points in $[4, -6] \cup \left[0, \frac{5}{2}\right]$.
Similarly, the two hyperbolic lines through p and the endpoints at infinity of ℓ_1 determine 4 intervals on $\mathbf{R} \cup \{\infty\}$ namely

$$(0, 2), \ [2, 4], \ (4, \infty), \ \text{and} \ [-\infty, 0]$$

The lines parallel to ℓ_1 correspond to the points in $[2, 4] \cup [-\infty, 0]$.



So, the lines through p parallel to both ℓ_1 and ℓ_2 correspond to the points in the intersection
$([2,4] \cup [-\infty, 0]) \cap \left([4,-6] \cup [0, \frac{5}{2}]\right) = [2, \frac{5}{2}] \cup [-\infty, -6] \cup \{0, 4\}$
(where $[-\infty, -6] = (-\infty, -6] \cup \{\infty\}$).