QUESTION
Apply the Gram-Schmidt process to the following vectors:
$(1, 2, 3, 4), (4, 3, 2, 6), (18, 11, 4, 22)$.
Explain your answer.

ANSWER The process gives

$$
\begin{aligned}
\mathbf{w}_1 &= (1, 2, 3, 4) \\
\mathbf{w}_2 &= (4, 3, 2, 6) - \frac{\{(4, 3, 2, 6).(1, 2, 3, 4)\}(1, 2, 3, 4)}{30} \\
&= (4, 3, 2, 6) - \frac{4}{3}(1, 2, 3, 4)
\end{aligned}
$$

For an orthonormal basis it is simpler to use
$3\mathbf{w}_2 = 3(4, 3, 2, 6) - 4(1, 2, 3, 4) = (8, 1, -6, 2)$
so

$$
\begin{aligned}
\mathbf{w}_3 &= (18, 11, 4, 22) - \frac{\{(18, 11, 4, 22).(8, 1, -6, 2)\}(8, 1, -6, 2)}{105} \\
&\quad - \frac{\{(18, 11, 4, 22).(1, 2, 3, 4)\}(1, 2, 3, 4)}{30} \\
&= (18, 11, 4, 22) - \frac{175}{105}(8, 1, -6, 2) - \frac{140}{30}(1, 2, 3, 4) \\
&= (18, 11, 4, 22) - \frac{5}{3}(8, 1, -6, 2) - \frac{14}{3}(1, 2, 3, 4) \\
3\mathbf{w}_3 &= (54, 33, 12, 66) - (40, 5, -30, 10) - (14, 28, 42, 56) \\
&= (0, 0, 0, 0).
\end{aligned}
$$

The explanation is that the three given vectors are linearly dependent.