

Question

The vectors \vec{OP} and \vec{OQ} are given respectively by the quantities $2\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. Find \vec{PQ} , and determine its length and direction cosines.

Answer

$$\vec{OP} = 2\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} \text{ and } \vec{OQ} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP} = (4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) - (2\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) = 2\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$$

$$\text{length of } \vec{PQ} = |\vec{PQ}| = \sqrt{2^2 + 5^2 + 7^2} = \sqrt{78}$$

$$\cos \alpha = \frac{x}{r} \quad \cos \beta = \frac{y}{r} \quad \cos \gamma = \frac{z}{r}$$

$$\Rightarrow \cos \alpha = \frac{2}{\sqrt{78}} \quad \cos \beta = \frac{-5}{\sqrt{78}} \quad \cos \gamma = \frac{7}{\sqrt{78}}$$