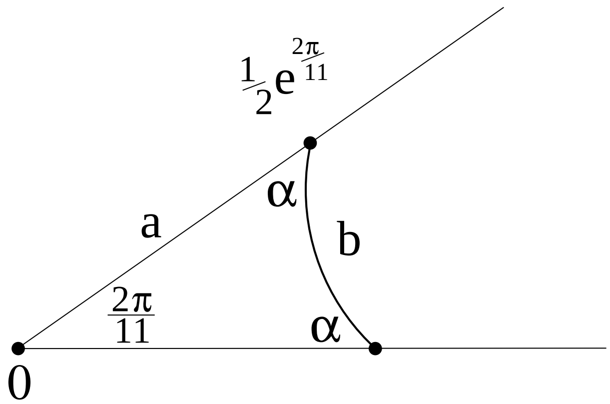
Question

Let P be a hyperbolic 11-gon in the Poincaré disc $\mathbf{D}$, with vertices at the points $\frac{1}{2}\exp\left(\frac{2\pi}{11}k\right)$ for $0 \leq k \leq 10$. Calculate the hyperbolic length of a side of P.

Answer



$a = \ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \ln(3)$

$\cosh(a) = \frac{1}{2}(e^a + e^{-a}) = \frac{1}{2}(3 + \frac{1}{3}) = \frac{5}{3}$

$\cosh(b) = \cosh(a)\cosh(a) - \sinh(a)\sinh(a)\cos\left(\frac{2\pi}{11}\right)$ (by lcI)

$\cosh(b) = \frac{25}{9} - \frac{16}{9}\cos\left(\frac{2\pi}{11}\right) = 1.2822$

$b = \ln(1.2822 + \sqrt{(1.2822)^2 - 1})$

$b = 0.73465$

Note that the sides of P all have the same length, since rotation by $\frac{2\pi}{11}$ takes P to P, and so the length of a side of P is $b = 0.73465$.