

Question

Solve the equation

(a) $(1 - z)^5 = z^5$

(b) $(z + 1)^6 = 56(z - 1)^6$

(c) $(5 + z)^5 - (5 - z)^5 = 0$

(d) $(z - \sqrt{3} + 2i)^6 + 64 = 0$

(e) $z^6 + z^4 + z^2 + 1 = 0$

Answer(a) $z = 0$ is not a solution.

$$\left(\frac{1-z}{z}\right)^5 = 1 \quad z = \frac{1}{1+w} \quad w = e^{\frac{2k\pi i}{5}} \quad h = 0, 1, \dots, 4$$

(b) $z = 1$ is not a solution

$$\left(\frac{z+1}{2(z-1)}\right)^6 = 1 \quad z = \frac{2w+1}{2w-1} \quad w = e^{2k\pi i/6} \quad k = 0, 1, \dots, 5$$

(c) $z = 5$ is not a solution

$$\left(\frac{5+z}{5-z}\right) z = \frac{5(w-1)}{w+1} \quad w = e^{\frac{2k\pi i}{5}} \quad k = 0, 1, \dots, 5$$

$$(d) \left(\frac{z - \sqrt{3} + 2i}{2i}\right)^6 \quad z = 2iw + \sqrt{3} - 2i \quad w = e^{\frac{2k\pi i}{6}} \quad k = 0, 1, \dots, 5$$

$$(e) z^6 + z^4 + z^2 + 1 = 0 \text{ iff } \frac{z^8 - 1}{z^2 - 1} = 0 \quad z^2 \neq 1 \text{ iff } z = e^{\frac{2k\pi i}{8}} \quad k = 1, 2, 3, 5, 6, 7$$