

Question

Solve the equation

$$(x + ia)^{2n} + (x - ia)^{2n} = (x^2 + a^2)^n$$

Answer

Assumption x and a are real $x + ia = re^{i\theta}$

$$(r^2 e^{2i\theta})^n + (r^2 e^{-2i\theta})^n = (r^2)^n$$

If $r \neq 0$

$$e^{2ni\theta} + e^{-2ni\theta} = 1$$

i.e. $\cos 2n\theta = \frac{1}{2}$

$$\theta = \left(\frac{2k}{n} + \frac{1}{6n} \right) \pi \quad k = 0, 1, 2, \dots, 2n - 1$$

If $r = 0$

then $a = 0$ so the equation becomes $x^{2n} + x^{2n} = x^{2n} \Rightarrow x = 0$