

Question

Show that a necessary and sufficient condition that the points denoted by z_1, z_2 and that the origin should form an equilateral triangle is

$$z_1^2 - z_1z_2 + z_2^2 = 0$$

Answer

O, z_1, z_2 form an equilateral triangle if and only if $z_2 = z_1 e^{\pm i\frac{2\pi}{3}}$

if and only if $z_2^3 = -z_1^3$ and $z_2 \neq z_1$

if and only if $(z_1 + z_2)(z_1^2 - z_1z_2 + z_2^2) = 0$ and $z_2 \neq z_1$

if and only if $(z_1^2 - z_1z_2 + z_2^2) = 0$